Algebra II Common Core State Standards

In Algebra I, students added, subtracted, and multiplied polynomials. In Algebra II, students divide polynomials with remainder, leading to the factor and remainder theorems. This is the underpinning for much of advanced algebra, including the algebra of rational expressions.

Themes from middle school algebra continue and deepen during high school. As early as grade 6, students began thinking about solving equations as a process of reasoning (6.EE.5). This perspective continues throughout Algebra I and Algebra II (A-REI). “Reasoned solving” plays a role in Algebra II because the equations students encounter can have extraneous solutions (A-REI.2).

In Algebra II, they extend the real numbers to complex numbers, and one effect is that they now have a complete theory of quadratic equations: Every quadratic equation with complex coefficients has (counting multiplicities) two roots in the complex numbers.

In grade 8, students learned the Pythagorean theorem and used it to determine distances in a coordinate system (8.G.6–8). In Geometry, students proved theorems using coordinates (G-GPE.4–7). In Algebra II, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of conic sections (e.g., G-GPE.1).

In Geometry, students began trigonometry through a study of right triangles. In Algebra II, they extend the three basic functions to the entire unit circle.

As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., S-ID.6). In a modeling context, they might informally fit an exponential function to a set of data, graphing the data and the model function on the same coordinate axes.

Mathematical Practices

While all of the mathematical practice standards are important in all three courses, four are especially important in the Algebra II course:

- **Construct viable arguments and critique the reasoning of others** (MP.3). As in geometry, there are central questions in advanced algebra that cannot be answered definitively by checking evidence. There are important results about *all* functions of a certain type — the factor theorem for polynomial functions, for example — and these require general arguments (A-APR.2). Deciding whether two functions are equal on an infinite set cannot be settled by looking at tables or graphs; it requires arguments of a different sort (F-IF.8).
- **Attend to precision** (MP.6). As in the previous two courses, the habit of using precise language is not only a tool for effective communication but also a means for coming to understanding. For example, when investigating loan payments, if students can articulate something like, “What you owe at the end of a month is what you owed at the start of the month, plus 1/12 of the yearly interest on that amount, minus the monthly payment,” they are well along a path that will let them construct a recursively defined function for calculating loan payments (A-SSE.4).
- **Look for and make use of structure** (MP.7). The structure theme in Algebra I centered on seeing and using the structure of algebraic expressions. This continues in Algebra II, where students delve deeper into transforming expressions in ways that reveal meaning. The example given in the standards — that $x4 – y4$ can be seen as the difference of squares — is typical of this practice. This habit of seeing sub-expressions as single entities will serve students well in areas such as trigonometry, where, for example, the factorization of $x4 – y4$ described above can be used to show that the functions cos4x – sin4x and cos2x – sin2x are, in fact, equal (A-SSE.2).

In addition, the standards call for attention to the structural similarities between polynomials and integers (A-APR.1). The study of these similarities can be deepened in Algebra II: Like integers, polynomials have a division algorithm, and division of polynomials can be used to understand the factor theorem, transform rational expressions, help solve equations, and factor polynomials.

- **Look for and express regularity in repeated reasoning** (MP.8). Algebra II is where students can do a more complete analysis of **sequences** (F-IF.3), especially arithmetic and geometric sequences, and their associated series. Developing recursive formulas for sequences is facilitated by the practice of abstracting regularity for how you get from one term to the next and then giving a precise description of this process in algebraic symbols (F-BF.2). Technology can be a useful tool here: Most Computer Algebra Systems allow one to model recursive function definitions in notation that is close to standard mathematical notation. And spreadsheets make natural the process of taking successive differences and running totals (MP.5).

The same thinking — finding and articulating the rhythm in calculations — can help students analyze mortgage payments, and the ability to get a closed form for a geometric series lets them make a complete analysis of this topic. This practice is also a tool for using difference tables to find simple functions that agree with a set of data.

Algebra II is a course in which students can learn some technical methods for performing algebraic calculations and transformations, but sense-making is still paramount (MP.1). For example, analyzing Heron’s formula from geometry lets one connect the zeros of the expression to the degenerate triangles. As in Algebra I, the modeling practice is ubiquitous in Algebra II, enhanced by the inclusion of exponential and logarithmic functions as modeling tools (MP.4). Computer algebra systems provide students with a tool for modeling all kinds of phenomena, experimenting with algebraic objects (e.g., sequences of polynomials), and reducing the computational overhead needed to investigate many classical and useful areas of algebra (MP.5).

Note: The standards addressed here are based on the CCSS document and the PARCC Mathematics Model Frameworks document that the state has indicated will be used as a guideline for EOC assessments. Instructional notes are taken from the Appendix A in the CCSS document.

★ = **specific modeling standards appear throughout the high school standards indicated by a star symbol.**

The Real Number System (N.RN)	
Clusters with Instructional Notes	**Standards**
Extend the properties of exponents to rational exponents.	**N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Quantities (N.Q)	
Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions*	**N.Q** .2 Define appropriate quantities for the purpose of descriptive modeling.
The Complex Number System (N.CN)	
Perform arithmetic operations with complex numbers. *Polynomials with real coefficients*	**N.CN.1** Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real. **N.CN.2** Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
Use complex numbers in polynomial identities and equations.	**N.CN.7** Solve quadratic equations with real coefficients that have complex solutions.
Seeing Structure in Expressions (A.SSE)	
Interpret the structure of expressions *Polynomial and rational*	**A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 + y^2)(x^2 - y^2)$*
Write expressions in equivalent forms to solve problems *Quadratic and exponential*	**A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* *c.* Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★
Arithmetic with Polynomials and Rational Expressions (A.APR)	
Understand the relationship between zeros and factors of polynomials	**A.APR.2** Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$. **A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
Use polynomial identities to solve problems	**A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ be used to generate Pythagorean triples.* **A.APR.5** (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal’s Triangle.
Rewrite rational expressions	**A.APR.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
Creating Equations* (A.CED)	
Create equations that describe numbers or relationships. *Equations using all available types of expressions, including simple root functions*	**A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Reasoning with Equations and Inequalities (A.REI)	
Clusters with Instructional Notes	**Standards**
Understand solving equations as a process of reasoning and explain the reasoning. *Simple radical and rational as well as Master linear; learn as general principle*	A.REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Solve equations and inequalities in one variable. *Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions.*	A.REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A.REI.4. Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
Solve systems of equations. *Linear-linear and linear-quadratic*	A.REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*
Represent and solve equations and inequalities graphically. *Combine polynomial, rational, radical, absolute value, and exponential functions*	A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
Interpreting Functions (F.IF)	
Understand the concept of a function and use function notation. *Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences*	F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for $n \geq 1$.
Interpret functions that arise in applications in terms of the context. *Emphasize selection of appropriate models*	F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* F.IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
Analyze functions using different representations. *Focus on using key features to guide selection of appropriate type of model function*	F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. F.IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. ᵗ b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.* F.IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
Building Functions (F.BF)	
Clusters with Instructional Notes	**Standards**
Build a function that models a relationship between two quantities *Include all types of functions studied*	F.BF.1 Write a function that describes a relationship between two quantities* a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
Build new functions from existing functions. *Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*	F.BF.3 Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. F.BF.4 Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.*

Linear, Quadratic, and Exponential Models* (F.LE)	
Construct and compare linear, quadratic, and exponential models and solve problems. *Logarithms as solutions for exponentials*	F.LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
Interpret expressions for functions in terms of the situation they model *Linear and exponential of form $f(x)=b^x+k$*	F.LE.5. Interpret the parameters in a linear or exponential function in terms of a context.
Trigonometric Functions (F.TF)	
Extend the domain of trigonometric functions using the unit circle	F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
Model periodic phenomena with trigonometric functions	F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
Prove and apply trigonometric identities	F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
Expressing Geometric Properties with Equations (G.GPE)	
Translate between the geometric description and the equation of a conic section	G.GPE.2 Derive the equation of a parabola given a focus and directrix.
Interpreting Categorical and Quantitative Data (S.ID)	
Summarize, represent, and interpret data on a single count or measurement variable.	S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
Making Inferences and Justify Conclusions (S.IC)	
Understand and evaluate random processes underlying statistical experiments	S.IC1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population. S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*
Make inferences and justify conclusions	S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. S.IC.6 Evaluate reports based on data.
Conditional Probability and the Rules of Probability (S.CP	
Understand independence and dependence and use them to interpret data *Link to data from simulations or experiments*	S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
Use the rules of probability to compute probabilities of compound events in a uniform probability model	S.CP.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

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