Algebra II Common Core State Standards

In Algebra I, students added, subtracted, and multiplied polynomials. In Algebra II, students divide polynomials with remainder, leading to the factor and remainder theorems. This is the underpinning for much of advanced algebra, including the algebra of rational expressions.

Themes from middle school algebra continue and deepen throughout Algebra I. As early as grade 6, students began thinking about solving equations as a process of reasoning (6.EE.5). This perspective continues throughout Algebra I and Algebra II (A-REI). "Reasoned solving" plays a role in Algebra II because the equations students encounter can have extraneous solutions (A-REI.2).

In grade 8, students learned the Pythagorean theorem and used it to determine distances in a coordinate system (8.G.6–8). In Geometry, students proved theorems using coordinates (G-GPE.4–7). In Algebra II, students will build on their understanding of distance in coordinate systems and draw on their growing command of algebra to connect equations and graphs of conic sections (e.g., G-GPE.1).

In Geometry, students began trigonometry through a study of right triangles. In Algebra II, they extend the three basic functions to all the unit circle.

As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., S-ID.6). In a modeling context, they might informally fit an exponential function to a set of data, graphing the data and the model function on the same coordinate axes. This practice of abstracting regularity for how you get from one term to the next and then giving a precise description of this process in algebraic notation is typical of this practice. This habit of seeing sub-expressions as single entities will serve students well in areas such as trigonometry, where, for example, the factorization of \( a^4 - 4 \) described above can be used to show that the functions \( 
\begin{align*}
\cos 4x & \quad \sin 4x \\
- \sin^2 2x & \quad \cos^2 2x
\end{align*}
\)
are, in fact, equal (A- SSE.2).

In the additional standards, call for attention to the structural similarities between polynomials and integers (A-APR.1). The study of these similarities can be deepened in Algebra II: Like integers, polynomials have a division algorithm, and division of polynomials can be used to understand the factor theorem, transform rational expressions, help solve equations, and factor polynomials.

The same thinking — finding and articulating the rhythm in calculations — can help students analyze mortgage payments, and the ability to get a feel for an algebraic expression may help them make a complete analysis of this topic. This practice is also a tool for using difference tables to find simple functions that agree with a set of data.

Algebra II is a course in which students can learn some technical methods for performing algebraic calculations and transformations, but sense-making is still paramount (MP.1). For example, analyzing Heron’s formula from geometry lets one connect the zeros of the expression to the degenerate triangles. As in Algebra I, the modeling practice is ubiquitous in Algebra II, enhanced by the inclusion of exponential and logarithmic functions as modeling tools (MP.4).

The complex number system (N.CN) is used throughout Algebra II. For example, \( \sqrt{-1} \) is used to generate Pythagorean triples.

Mathematical Practices

- Construct viable arguments and critique the reasoning of others (MP.3).
- Attend to precision (MP.6). As in the previous two courses, the habit of using precise language is not only a tool for effective communication but also a means for coming to understanding. For example, when investigating loan payments, if students can articulate something like, “What you owe at the end of a month is what you owed at the start of the month, plus 1/12 of the yearly interest on that amount, minus the monthly payment,” they are well along a path that will let them construct a recursively defined function for calculating loan payments (A- SSE.4).
- Look for and make use of structure (MP.7). The structure theme in Algebra I centered on seeing and using the structure of algebraic expressions. This continues in Algebra II, where students delve deeper into transforming expressions in ways that reveal meaning. The example given in the standards — that \( e^4 - 4 \) can be seen as the difference of squares — is typical of this practice. This habit of seeing sub-expressions as single entities will serve students well in areas such as trigonometry, where, for example, the factorization of \( a^4 - 4 \) described above can be used to show that the functions \( \cos 4x = \sin 4x \\
- \sin^2 2x = \cos^2 2x 
\)
are, in fact, equal (A- SSE.2).

- Look for and express regularity in repeated reasoning (MP.8). Algebra II is where students can do a more complete analysis of sequences (F- IF.3), especially arithmetic and geometric sequences, and their associated series. Developing recursive formulas for sequences is facilitated by the practice of abstracting regularity for how you get from one term to the next and then giving a precise description of this process in numerical and algebraic notation. The work with sequence and series in Algebra II will be used to generate Pythagorean triples.

A.SSE.3
Interpret the structure of expressions

- Polynomial and rational

A.SSE.3 C
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- Use the properties of exponents to transform expressions for exponential functions. For example the expression \( \frac{1.15t^t}{1.02^t} \) can be rewritten a \( (\frac{1.15}{1.02})^t \) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A.SSE.4
Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), use the formula to solve problems. For example, calculate mortgage payments.

A.APR.1
Understand the relationship between zeros and factors of polynomials

- Use polynomial identities to solve problems

A.APR.3
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.APR.4
Prove polynomial identities and use them to describe simple relationships. For example, the polynomial identity \( (x+y)^2 = x^2 + 2xy + y^2 \) is used to generate Pythagorean triples.

A.APR.5
Know and apply the Binomial Theorem for the expansion of \((x + y)^n\) in powers of\(x\) and \(y\) for a positive integer \(n\), where \(x\) and \(y\) are any numbers, with coefficients determined for example by Pascal’s Triangle.

A.APR.6
Rewrite rational expressions in different forms; write \( ax + by + c\) in the form \( a(x + y) + c\), where \(a\), \(b\), \(c\), and \(x\) and \(y\) are polynomials with the degree of \(x\) less than the degree of \(b(x)\), using inspection, long division, or for the more complicated examples, a computer algebra system.

Create equations that describe numbers or relationships.

A.CED.1
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
Clusters with Instructional Notes

Standards

Reasoning with Equations and Inequalities (A.REI)

A.REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable.

A.REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.4. Solve quadratic equations in one variable.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$, take square roots, completing the square, quadratic formula and factoring, as appropriate to the initial form of the equation).

Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers $a$ and $b$.

A.REI.4.B. Solve quadratic equations with real coefficients that have complex solutions.

A.REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

A.REI.11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Interpreting Functions (F.IF)

F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = 0$, $f(1) = 1$, $f(n) = f(n-1) + f(n-2)$ for $n > 1$.

F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the situation they model. Include points of intersection, intervals where the function is increasing or decreasing, maximum or minimum values, and end behavior.

F.IF.5. Relate the average rate of change of a function (interpreted as a rate of change) over a specified interval to the slope of the graph as a point of change.

F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for complex cases.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F.IF.8. Write a function defined in terms of another function and provide examples.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these properties to the structure of the expression.

c. Recognize the effects of exponentiation on functions; graph exponential and logarithmic functions, showing key features.

F.IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions (F.BF)

F.BF.1. Write a function that describes a relationship between two quantities. Include simple rate of change as well as linear and exponential functions; emphasise common effect of each transformation across function types.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these properties to the structure of the expression.

F.BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

F.SCP.6. Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.

F.SCP.7 Apply the Addition Rule, $P(A\text{ or } B) = P(A) + P(B) - P(A\text{ and } B)$, and interpret the answer in terms of the model.